

# 2.5層水温躍層中の波と周期的な風 強制に対する応答

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# はじめに

- 海洋が大気の変動に対してどのように応答するかは、気候変動を考える上で重要な問題である。
- もっとも単純な静止海洋を考えれば、渦位は $\beta$ によって支配され、すべてのロスビー波は西方に伝播する。しかし、海洋循環が存在する場合、渦位傾度の向きは深さにより異なり、波の伝播は単純に西方というわけではなくなる。特に、熱的強制と風の強制の違いなどが一つの焦点になる。
- 本セミナーでは、2.5層水温躍層を基本場とし、前半では、その世界での波の性質について解説し、後半では、全域で振動する風強制を与えた時の波の発生と伝播について述べる。(前半の話は10年以上前の話になるが、講義の続きの意味もあって紹介する。後半は1年ほど前の研究である。)

Kubokawa, A. and M. Nagakura (2002): Linear planetary wave dynamics in a 2.5-layer ventilated thermocline model. J. Mar. Res., 60, 367-404.

Kubokawa, A. (2013): Linear Response of a Ventilated Thermocline to Periodic Wind Forcing. J. Phys. Oceanogr., 43, 1811-1820

## 2.5層水温躍層

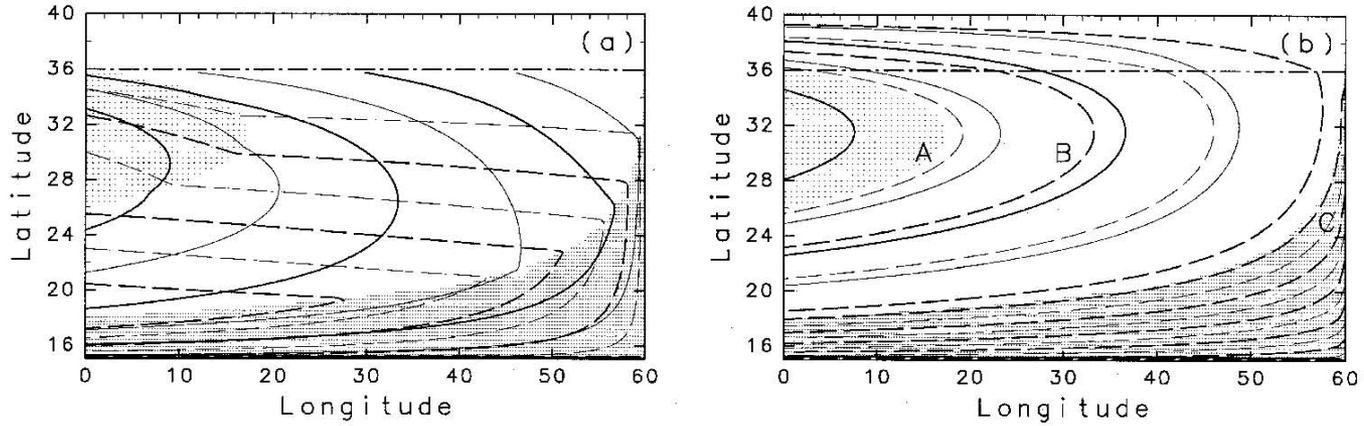


Figure 1. The solution of the ventilated thermocline model used as the basic state of the present study: (a) upper layer, and (b) second layer. Solid lines denote the pressure (contour interval is  $0.5 \times 10^3 \text{ Kg m}^{-1} \text{ s}^{-2}$ ) and dashed lines denote the potential thickness (contour interval is  $7.5 \times 10^3 \text{ m s}$ ). Heavy shade denotes the shadow zone where the second layer is at rest, and light shade denotes the RY pool where the second layer potential vorticity is homogenized. The letters A, B and C denote locations where local eigenvalues are calculated.

$$-fv_j = -\frac{1}{a \cos \theta} \frac{\partial p_j}{\partial \lambda}, \quad fu_j = -\frac{1}{a} \frac{\partial p_j}{\partial \theta} \quad (2.1)$$

$$\frac{\partial h_j}{\partial t} + \frac{1}{a \cos \theta} \left\{ \frac{\partial}{\partial \lambda} h_j u_j + \frac{\partial}{\partial \theta} h_j v_j \cos \theta \right\} = -(2-j)w_e - (-1)w_d \quad (2.2)$$

$$p_2 = \gamma_2(h_1 + h_2), \quad p_1 = p_2 + \gamma_1 h_1 \quad (2.3)$$

# 擾乱の方程式

$$\frac{\partial}{\partial t} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \frac{1}{a \cos \theta} \mathbf{A} \frac{\partial}{\partial \lambda} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \frac{1}{a} \mathbf{B} \frac{\partial}{\partial \theta} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \mathbf{C} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \mathbf{W} \quad (2.4)$$

where  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are  $2 \times 2$  matrices and  $\mathbf{W}$  is a vector:

$$\mathbf{A} = \begin{pmatrix} U_2 - \frac{\beta}{f^2} \gamma_1 H_1 & \gamma_2 \left( \frac{H_1}{f} \right)_y \\ -\frac{\beta}{f^2} \gamma_1 H_1 & -\frac{\beta}{f^2} \gamma_2 H_1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} V_2 & -\frac{\gamma_2}{f} H_{1x} \\ 0 & 0 \end{pmatrix}, \quad (2.5)$$

$$\mathbf{C} = \begin{pmatrix} -\frac{\beta}{f} V_1 & 0 \\ -\frac{\beta}{f^2} \gamma_1 H_{1x} & -\frac{\beta}{f^2} \gamma_2 H_x \end{pmatrix}, \quad \mathbf{W} = \begin{pmatrix} -w_e + w_d \\ -w_e \end{pmatrix}. \quad (2.6)$$

Here,  $\beta = a^{-1} df/d\theta$  and subscripts,  $x$  and  $y$ , denote the zonal derivative and meridional derivative, e.g.,

$$H_{1x} = \frac{1}{a \cos \theta} \frac{\partial H_1}{\partial \lambda}, \quad H_{1y} = \frac{1}{a} \frac{\partial H_1}{\partial \theta}.$$

# 局所的な波の性質

波が進む方向を $\phi$ とする

$$\frac{\partial}{\partial t} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \mathbf{D} \frac{\partial}{\partial s} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \mathbf{W}, \quad (3.2)$$

$$s = \frac{x}{\cos \phi} = \frac{y}{\sin \phi}$$

$$\mathbf{D} = \mathbf{A} \cos \phi + \mathbf{B} \sin \phi.$$

$\mathbf{D}$ の固有値が波の位相速度を表す。

$$c_{\pm} = \frac{1}{2} \{ (d_{11} + d_{22}) \pm \sqrt{(d_{11} - d_{22})^2 + 4d_{12}d_{21}} \},$$

群速度は

$$c_{gx} = \frac{\partial c}{\partial(\cos \phi)}, \quad c_{gy} = \frac{\partial c}{\partial(\sin \phi)}.$$

# プール領域での解

Non-Doppler Shift Mode → N-mode  
 Advective mode → A-mode

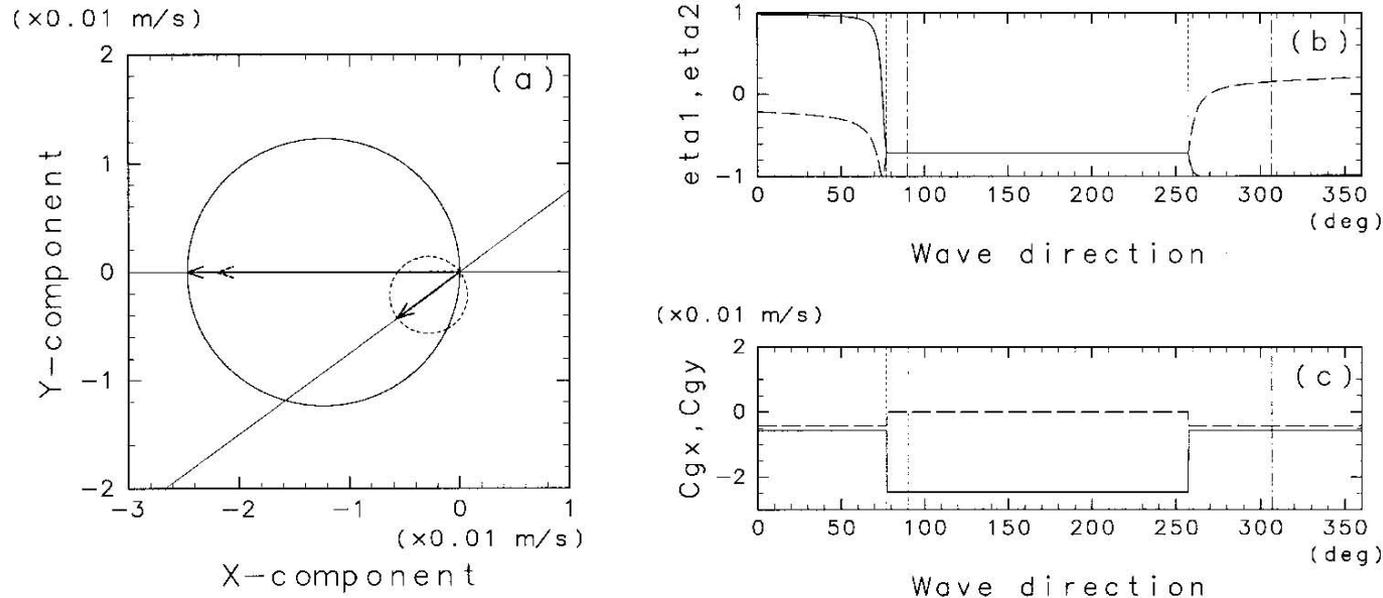
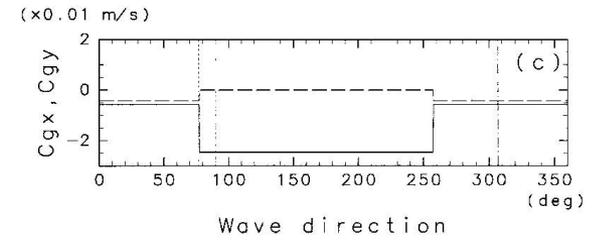
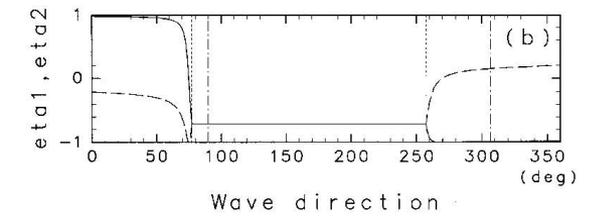
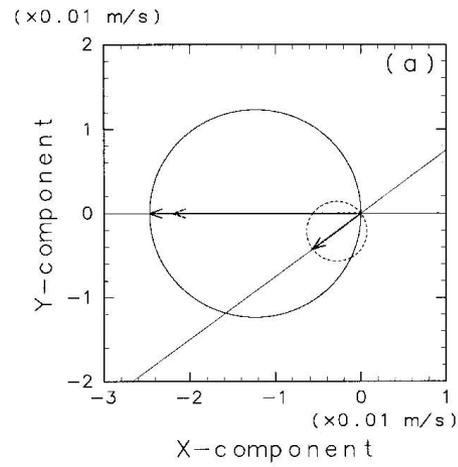
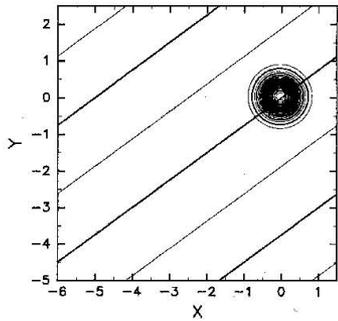


Figure 2. Eigenvalue solution in the RY pool (location A in Fig. 1: longitude =  $15^\circ$ , latitude =  $30^\circ$ ) as a function of the direction of wavenumber vector: (a) two-dimensional diagram of the phase velocity vector,  $(c(\varphi)\cos \varphi, c(\varphi)\sin \varphi)$ , (b) eigenvectors (vertical structure) and (c) group velocities. In panel (a), the solid curve corresponds to the *N*-mode, the dashed curve to the *A*-mode. The westward longer arrow (solid) is the maximum phase velocity vector of the *N*-mode which is identical to the group velocity; the westward shorter arrow (dotted) is the group velocity without the basic flow. The southwestward arrow denotes the basic current velocity in the second layer. In panel (b), the solid line and the dashed line denote  $\eta_1$  and  $\eta_2$ , respectively, and their amplitudes are normalized as satisfying  $\sqrt{\eta_1^2 + \eta_2^2} = 1$ . In panel (c), the solid line and the dashed lines denote  $c_{gx}$  and  $c_{gy}$ , respectively. The vertical dotted lines and dash-dotted lines in panel (b) and (c) denote the wave directions where the *N*-mode and the *A*-mode are separated, and those where the solutions are stationary ( $c = 0$ ), respectively.

# プール領域での解

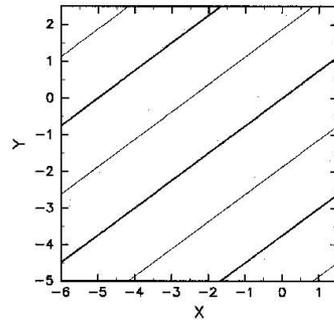


eta1; t= .0



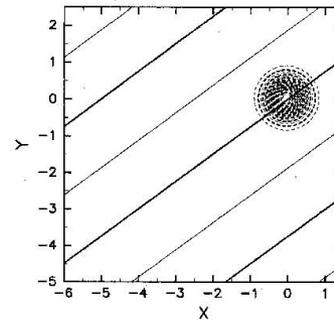
CONTOUR INTERVAL = 5.000E-2

eta2; t= .0



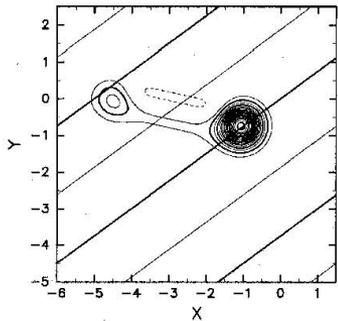
CONTOUR INTERVAL = 5.000E-2

q2; t= .0



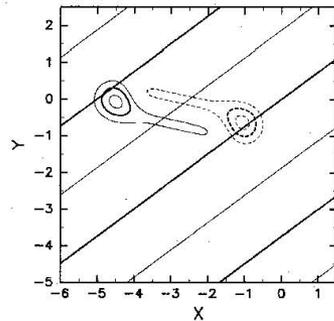
CONTOUR INTERVAL = 5.000E-2

eta1; t=4.0



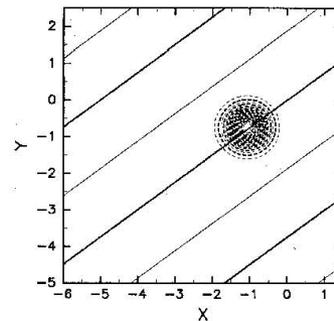
CONTOUR INTERVAL = 5.000E-2

eta2; t=4.0



CONTOUR INTERVAL = 5.000E-2

q2; t=4.0



CONTOUR INTERVAL = 5.000E-2

# 通気領域での解

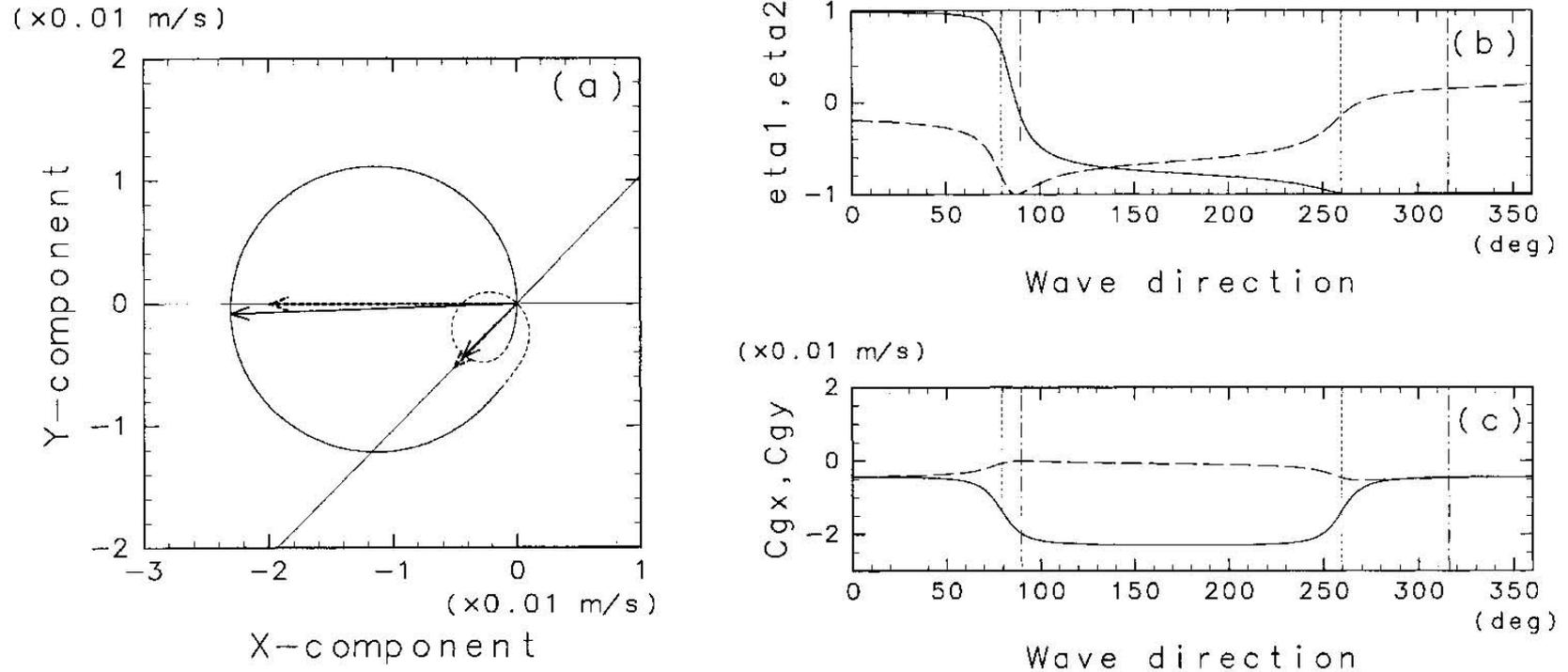
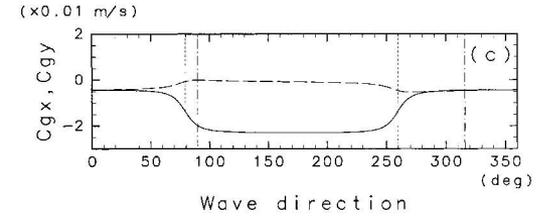
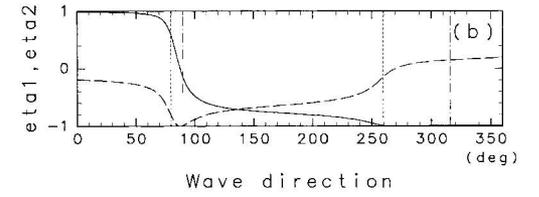
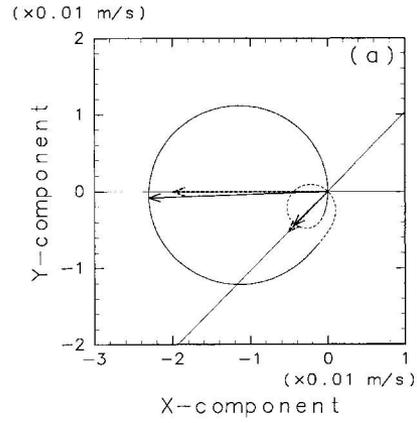
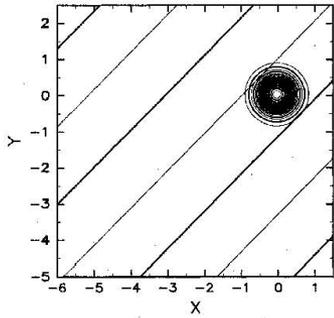


Figure 4. Same as Figure 2 but for the ventilated zone (location B in Figure 1: longitude =  $30^\circ$ , latitude =  $30^\circ$ ). In panel (a), the westward and southwestward solid arrows denote the typical group velocity vectors of the  $N$ -mode and the  $A$ -mode, respectively, and the southwestward dotted arrow denotes the basic flow velocity in the second layer.

# 通気領域での解

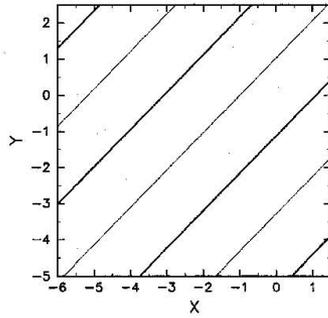


eta1; t= .0



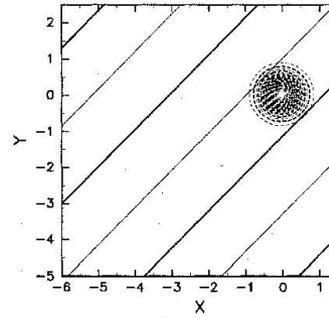
CONTOUR INTERVAL = 5.000E-2

eta2; t= .0



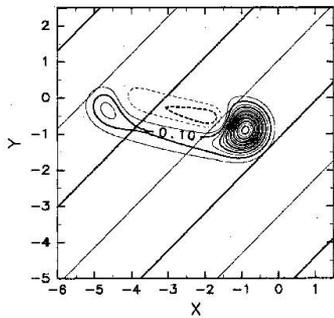
CONTOUR INTERVAL = 2.500E-2

q2; t= .0



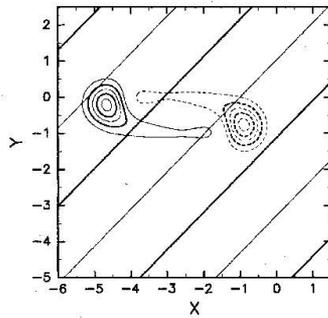
CONTOUR INTERVAL = 5.000E-2

eta1; t=4.0



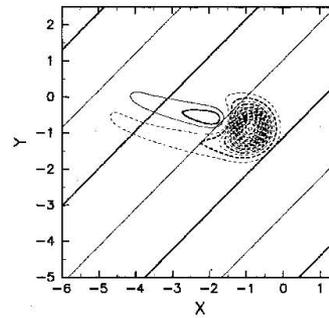
CONTOUR INTERVAL = 5.000E-2

eta2; t=4.0



CONTOUR INTERVAL = 2.500E-2

q2; t=4.0



CONTOUR INTERVAL = 5.000E-2

# 影領域での解

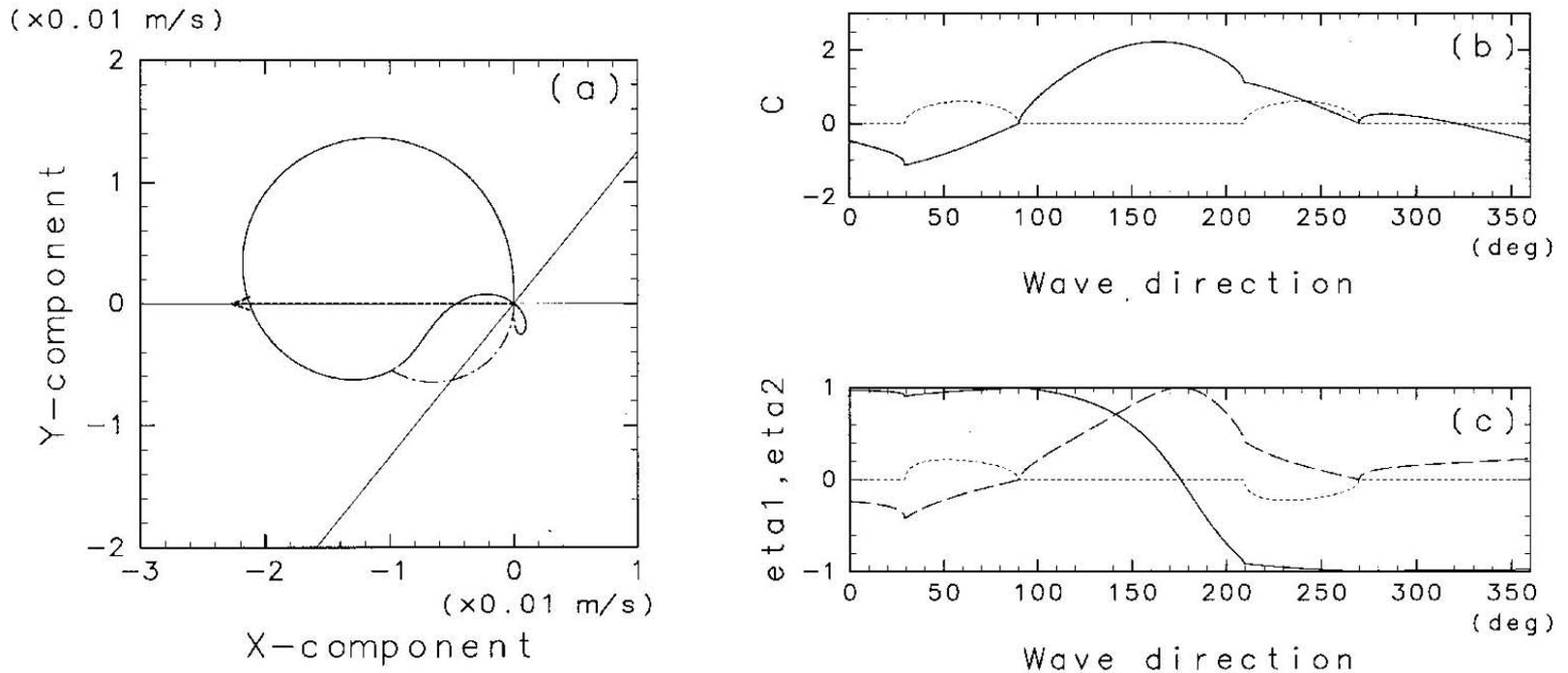


Figure 7. Eigenvalue solution in the shadow zone (location C in Fig. 1: longitude =  $58^\circ$ , latitude =  $25^\circ$ ) as a function of the direction of the wavenumber vector: (a) two-dimensional diagram of phase velocity vector, (b) eigenvalue  $c(\varphi)$  and (c) eigenvectors (vertical structure). In panel (a), at the portion denoted by the dash-dotted line, the eigenvalue  $c$  is complex. The westward dotted arrow is the group velocity without the basic flow. In panel (b), the dotted line denotes the imaginary part of  $c$ . In panel (c), the solid, dashed, and dotted lines denote  $\eta_1$ , the real part of  $\eta_2$  and the imaginary part of  $\eta_2$ , respectively, where  $\eta_1$  is assumed to be real and their amplitudes are normalized as satisfying  $\sqrt{|\eta_1|^2 + |\eta_2|^2} = 1$ .

# 波の特徴的な構造

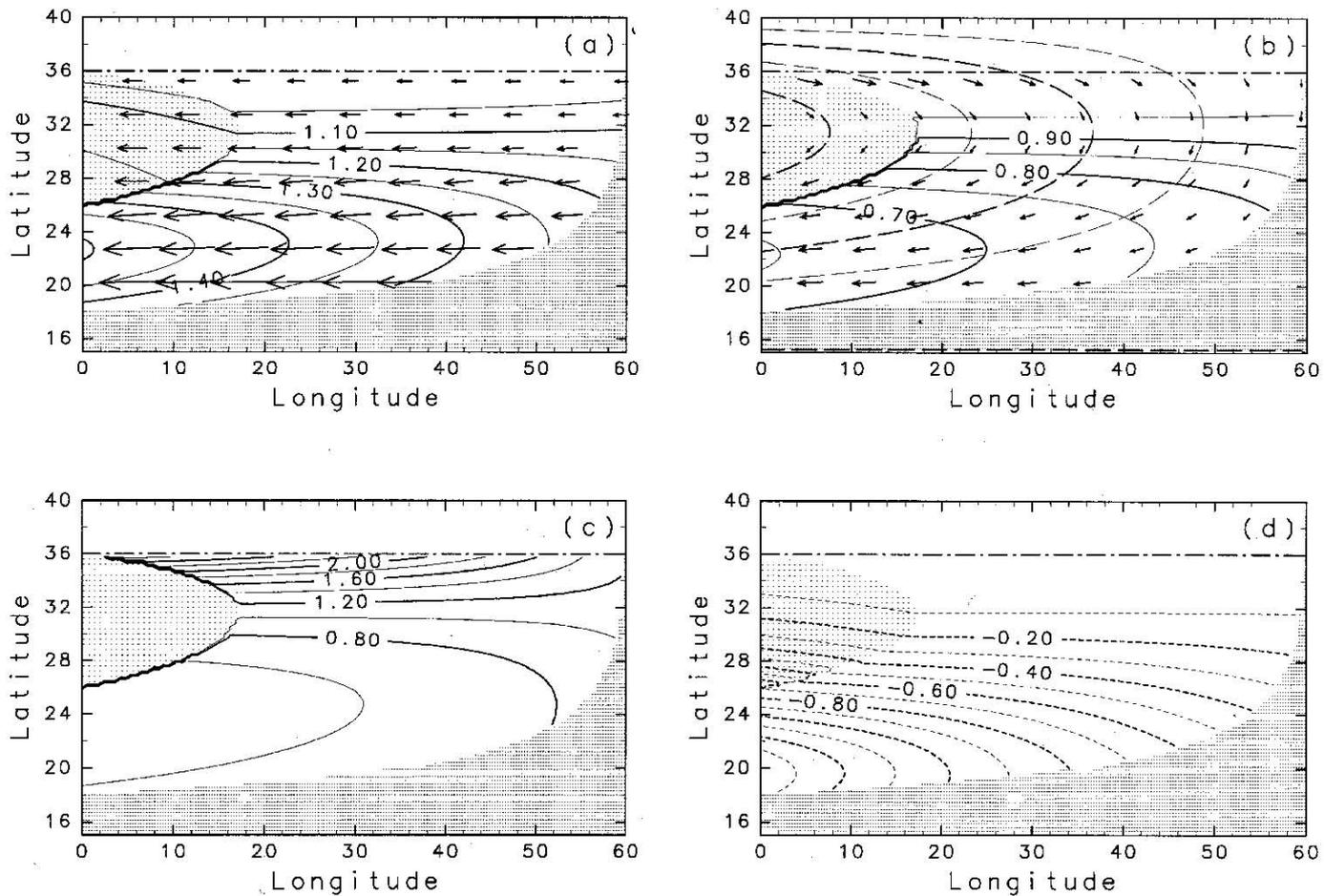


Figure 11. Gyre scale distribution of wave properties: (a) typical group velocity of the  $N$ -mode, (b) typical group velocity of the  $A$ -mode, (c) typical vertical structure,  $\eta_2/\eta_1$ , of the  $N$ -mode, and typical vertical structure of the  $A$ -mode. Solid contours in (a) and (b) are the ratios of the group velocities to  $c_1$  and the second layer advection speed, respectively. The dashed contours in (b) are the second layer stream lines,  $P_2$ .

# 通気領域の初期擾乱の発展

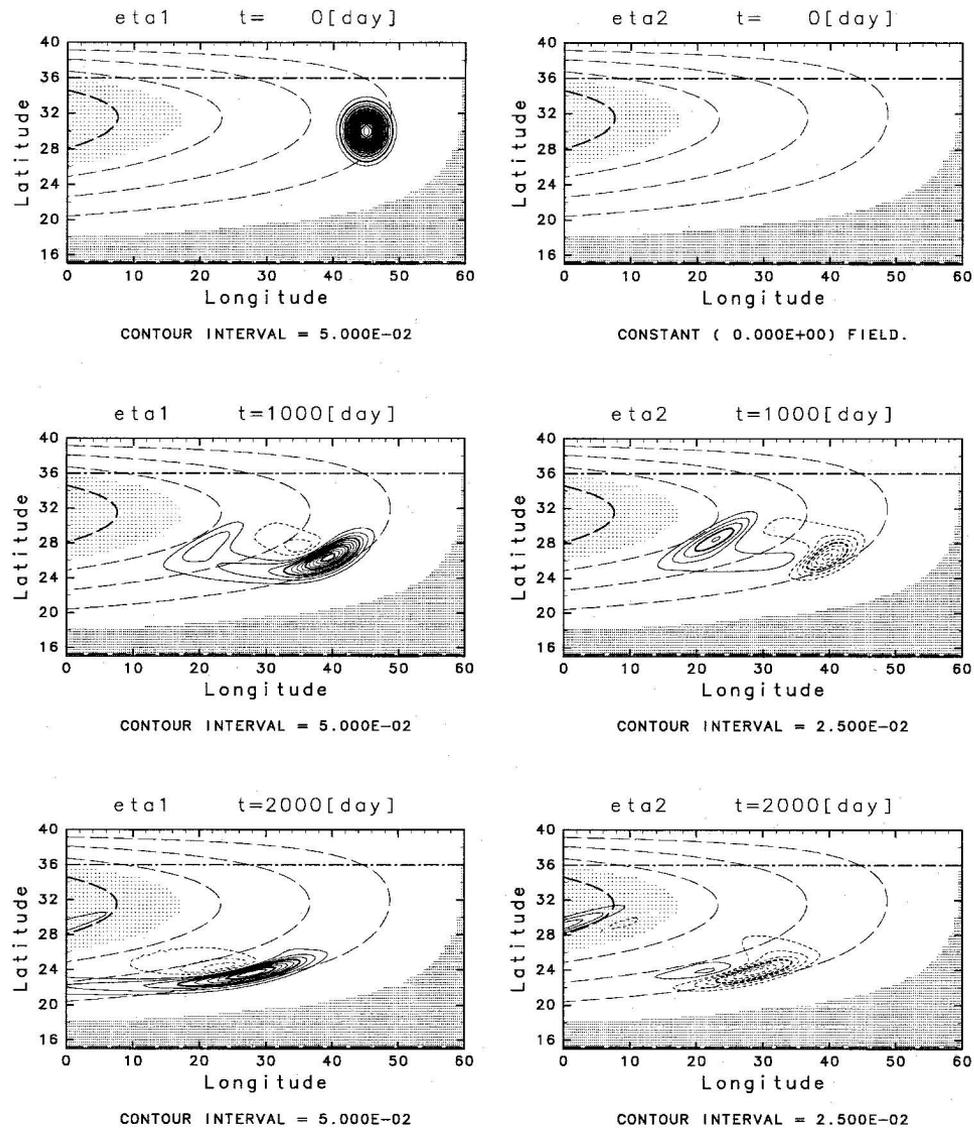


Figure 12. Gyre-scale evolution of an initial disturbance given only in  $\eta_1$ :  $\eta_1$  (left column) and  $\eta_2$  (right column). The amplitude is normalized by that of the initial disturbance, and contour intervals are denoted below each panel.

# 通気領域への強制への応答

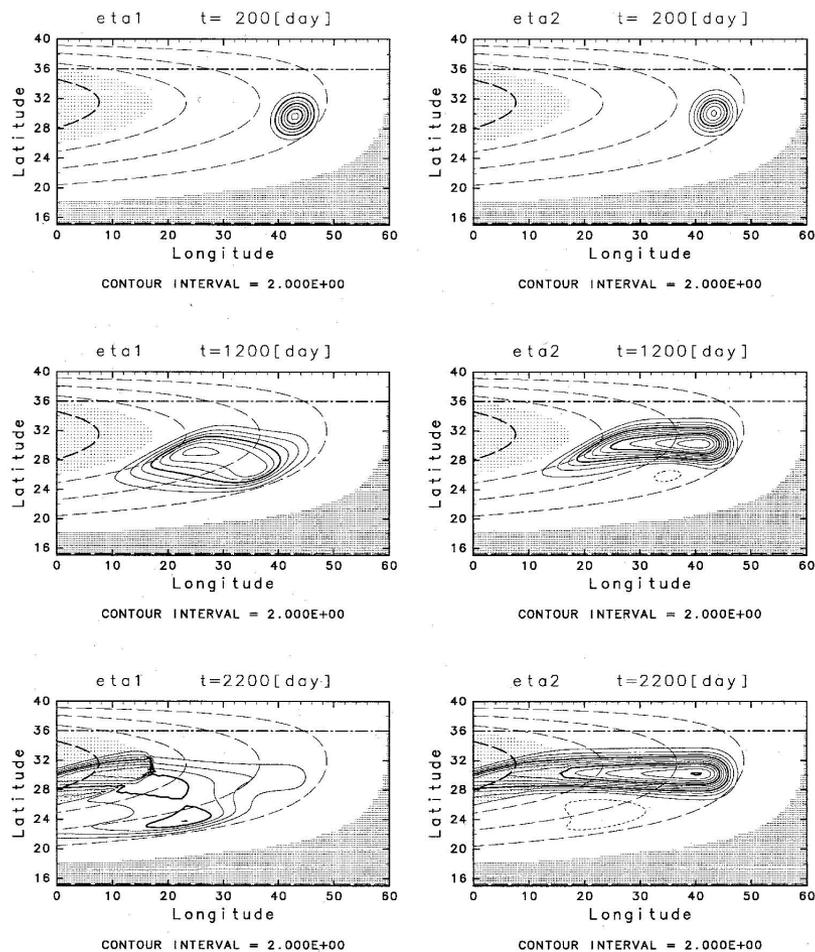


Figure 14. Gyre-scale evolution of disturbances generated by  $w_e$ -forcing which is turned on at  $t = 0$ :  $\eta_1$  (left column),  $\eta_2$  (right column). The forcing has the same lateral structure as the initial disturbance in Figure 11, and the magnitude of the forcing is  $10^{-6} \text{ m s}^{-1}$ . The contour intervals [unit is m] are denoted below each panel.

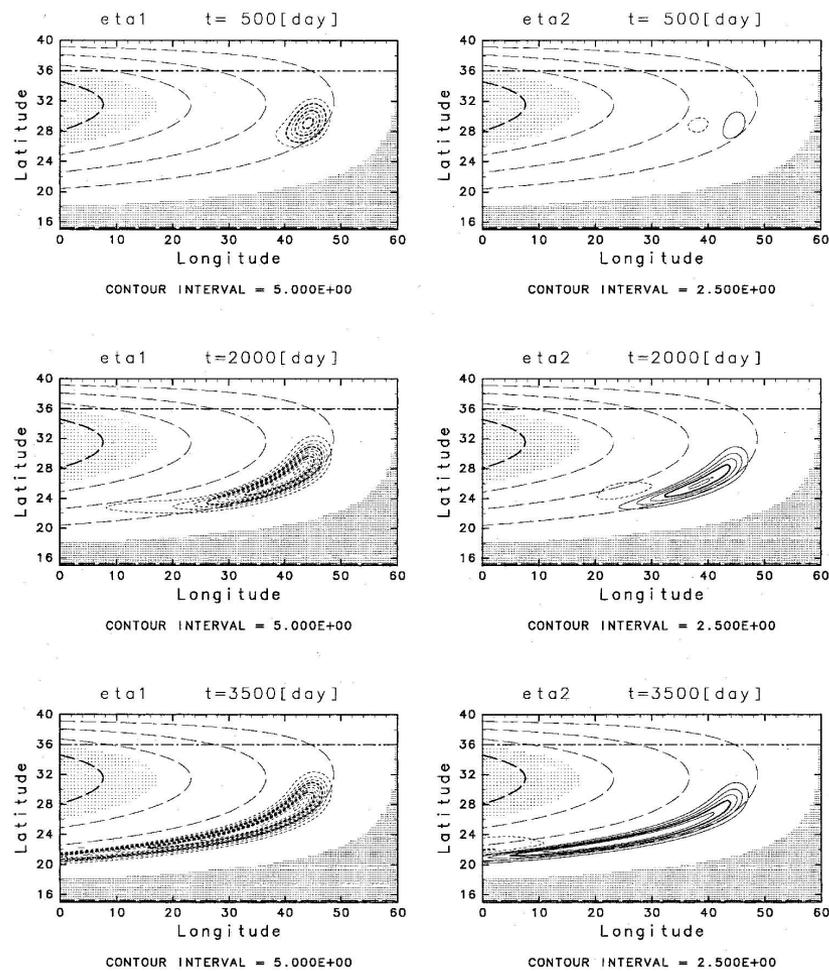


Figure 15. Same as Figure 14 but for  $w_a$ -forcing.

# 有限波長の影響

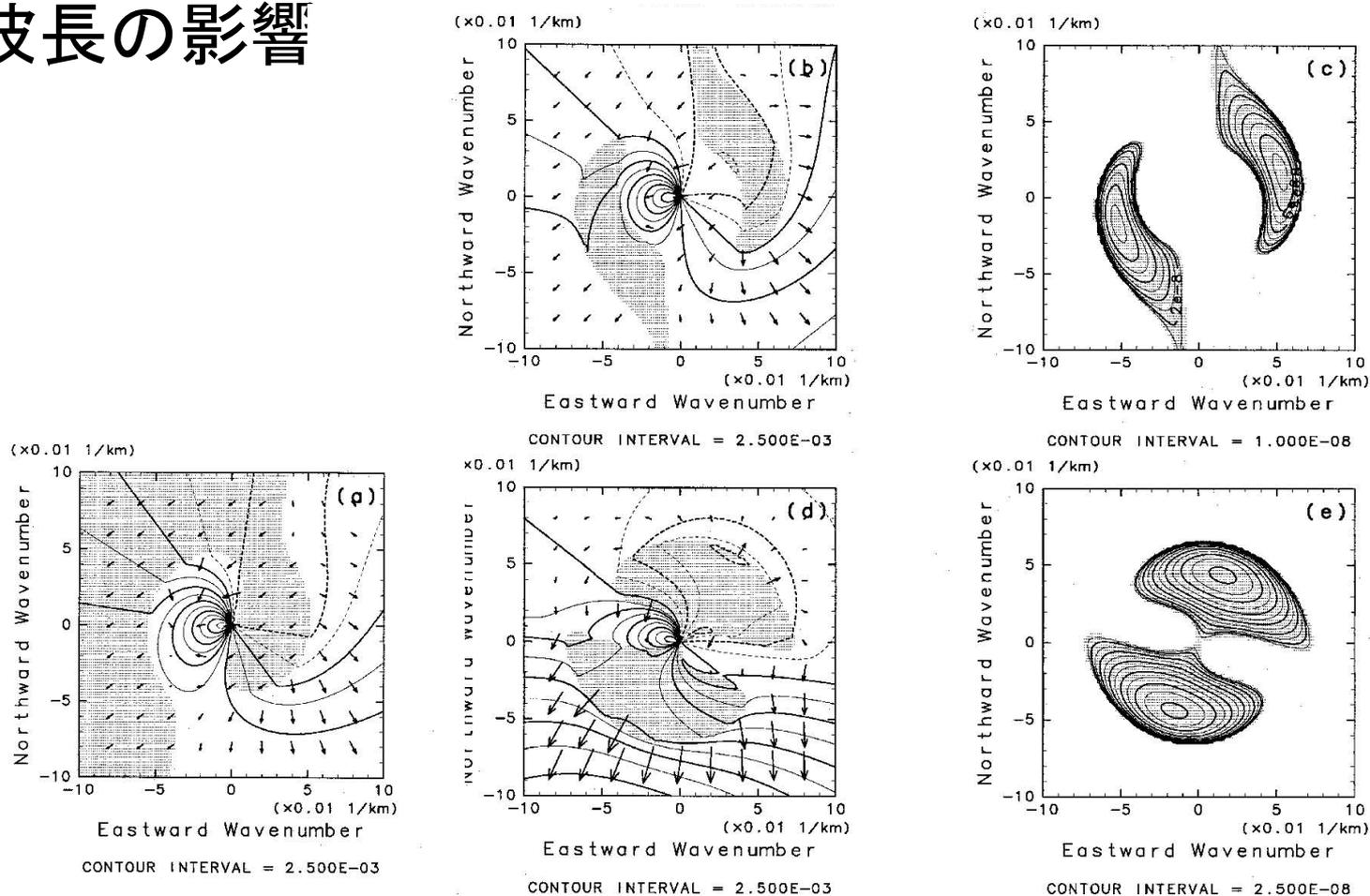


Figure 16. Phase speeds, group velocities [ $\text{m s}^{-1}$ ] and growth rate [ $\text{s}^{-1}$ ] in the  $(k, l)$  plane at locations A (RY pool), B (ventilated zone) and C (shadow zone) in Figure 1b: (a) phase speed (contour) and group velocity (arrow) at location A where the waves are stable, (b) phase speed (contour) and group velocity (arrow) at location B, (c) growth rate at location B, (d) phase speed (contour) and group velocity (arrow) at location C, (e) growth rate at location C. Shade in (a) denotes the region where the wave is non-dispersive (group velocity coincides with the second layer current velocity), and the shade in the other panels denotes the unstable region.

# 有限波長の波の性質

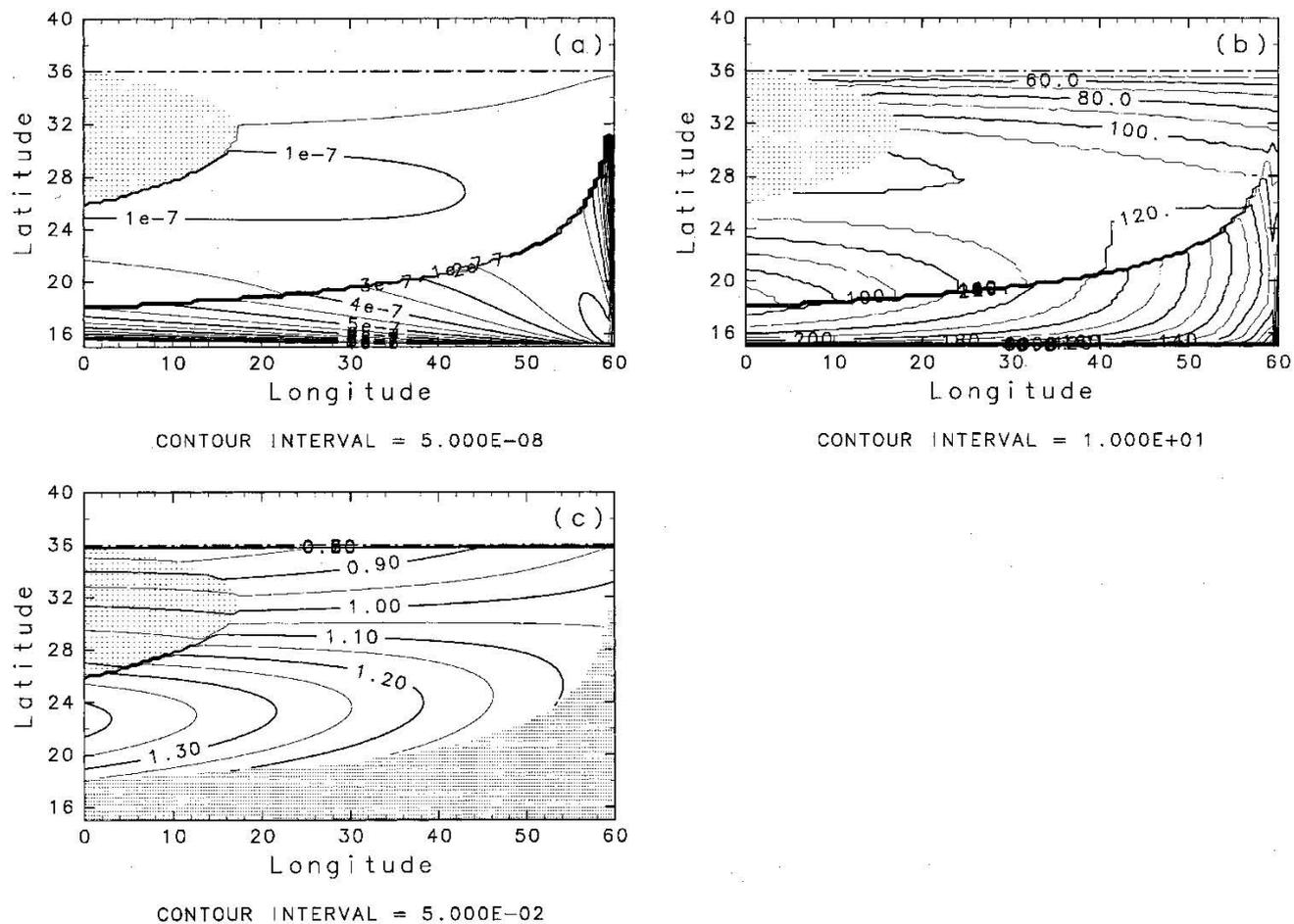


Figure 17. Gyre-scale distributions of wave properties with finite wavelength: (a) maximum growth rate  $[\text{s}^{-1}]$ , (b) wavelength of the most unstable wave [km], and (c) westward phase speed of the  $N$ -mode with 628 km wavelength relative to  $c_1$ .

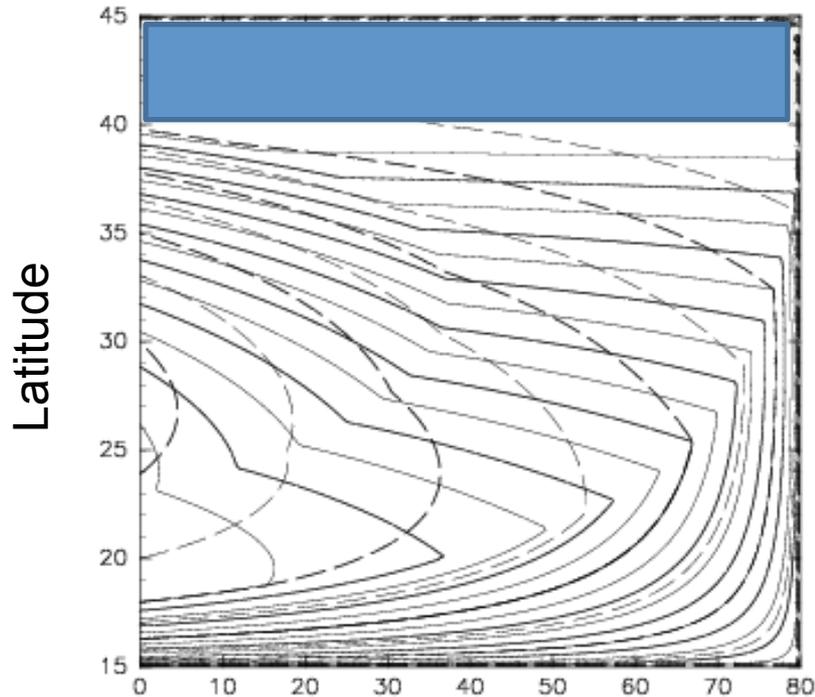
## 前半のまとめ

- プール領域では、N-mode、A-modeともに非分散であり、N-modeは静止海洋における第1傾圧モードよりも速く西へ伝波、A-modeは、2層目の流れで流される。鉛直構造は、波数ベクトルの向きに若干依存するため、任意の孤立した初期擾乱を考えたとき、両者は完全には分離しない。
- 通気領域では、両波は分散性を持つ。N-modeの最大群速度はほぼ西だが、若干南向きで、静止海洋における第1傾圧モードよりも速い。A-modeの代表的な群速度は、2層目の移流方向より若干右向きで、移流速度よりは遅い。
- 影領域では、長波も常に不安定である。他方、通気領域では、少なくともoutcropの緯度が一定の場合には、長は擾乱に対して安定である(証明可、ここでは省略)
- 風強制に対しては、初期にはA-modeも発生するが、基本的にはN-mode応答であり、東西一様な応答となる。熱(鉛直混合)強制に対しては、初期には弱いN-modeの発生も見られるが、主にはA-modeであり、時間がたつと影響は2層目の深度の等値線に完全に一致する。
- 各領域の波の波長依存性も調べた。プール領域では、2層目に移流されるだけの領域と、波動的な部分に分かれる。通気領域は、有限波長の波に対しては不安定になる。ただし、影領域に比べると不安定性は小さい。

次に、2.5層通気水温躍層モデルを用い、このような波動を持つ循環の、大規模な周期的な風応力変動に対する応答を見てみよう。

## 2.5層通気水温躍層

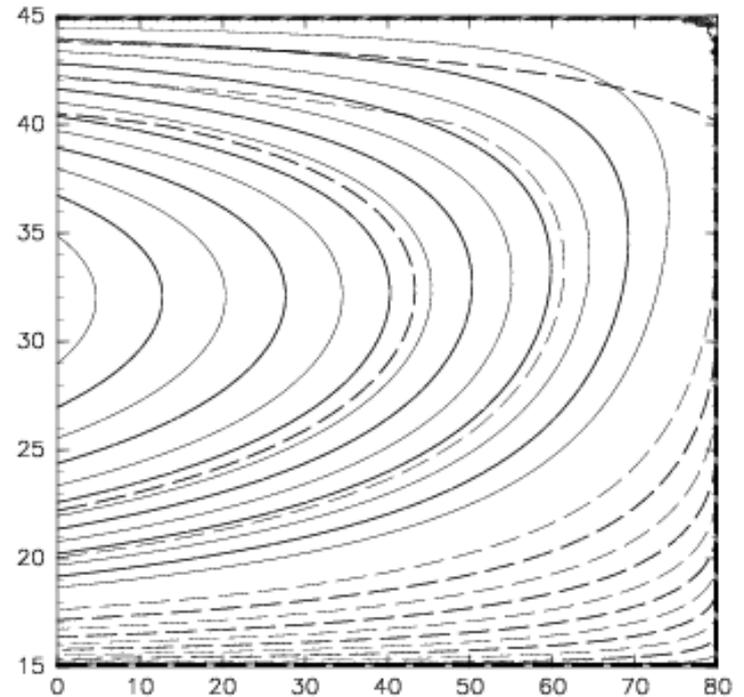
Outcropは40度、H2は東岸で750m、深さのコンタ間隔は25m



Longitude

H1 : 上層の厚さ (実線)

P1 : 上層の等圧線 (破線)



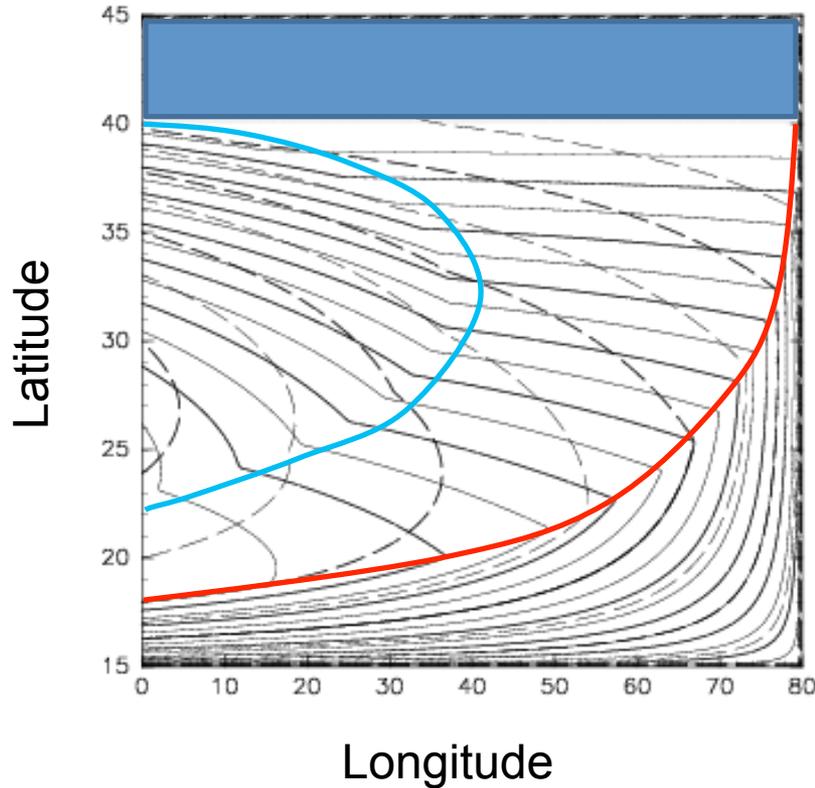
Longitude

H2 : 2層目下面の深さ (実線)

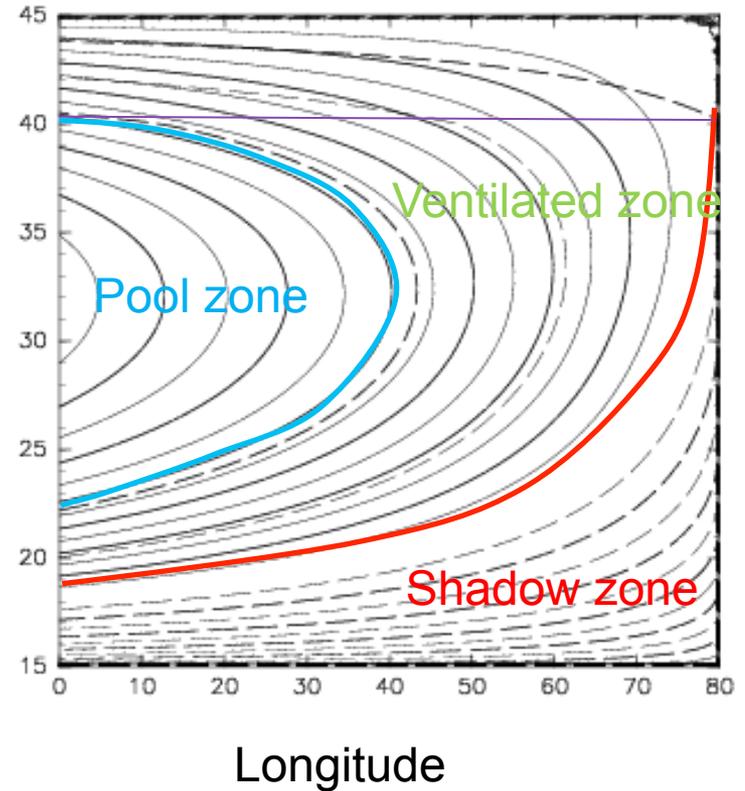
Q2 : 2層目のポテンシャル厚さ、  
(H2-H1) / f (破線)

## 2.5層通気水温躍層

Outcropは40度、H2は東岸で750m、深さのコンタ間隔は25m



H1 : 上層の厚さ (実線)  
P1 : 上層の等圧線 (破線)



H2 : 2層目下面の深さ (実線)  
Q2 : 2層目のポテンシャル厚さ、  
(H2-H1) / f (破線)

## 定式化：擾乱場

振動するエクマンパンピングを重ねる  $w_e(y, t) = W_e(y) + \hat{w}_e e^{i\sigma t}$

密度界面の深さも振動：

$$h_j(x, y, t) = H_j(x, y) + \eta_j(x, y) e^{i\sigma t}$$

方程式は、

$$i\sigma\eta_2 - \frac{\partial}{\partial x} \left[ \frac{\beta}{f^2} (\gamma_1 H_1 \eta_1 + \gamma_2 H_2 \eta_2) \right] = -\hat{w}_e$$

$$i\sigma(\eta_2 - \eta_1) + \gamma_2 J(H_2, (\eta_2 - \eta_1)/f) + \gamma_2 J(\eta_2, Q_2) = 0$$

ここで、 $\gamma_1, \gamma_2$  は密度界面に働く有効重力

東岸での境界条件は  $\eta_1(x_e, y) = \eta_2(x_e, y) = 0$

outcropは固定、したがって、 $\eta_1(x, y_1) = 0$

shadow zone と ventilated zone, ventilated zone と pool zoneの境界は自由境界である。したがって、自由境界の直角方向の変位は

$$i\sigma\xi + U_2 \cdot \nabla\xi = \mathbf{n} \cdot \hat{\mathbf{u}}_2 = -\frac{\gamma_2}{f} \mathbf{k} \cdot (\mathbf{n} \times \nabla\eta_2) \quad |$$

この  $\xi$  が自由境界の両側で一致するように西側  $\eta_2$  を決めることにより接続する

# 解き方

擾乱の方程式の特性曲線は、第1式は西向きで第2式では等Q2線、もしくは、等H2線に沿っている。また、接続条件でつなぐ自由境界も等Q2線かつ等H2線である。そこで、座標を x-y から Q2-y 座標に移す。(pool zoneはH2-y 座標)  
方程式は

$$\frac{i\sigma\eta_2}{\left(\frac{\partial Q_2}{\partial x}\right)_y} - \left(\frac{\partial}{\partial Q_2} \left[ \frac{\beta}{f^2} (\gamma_1 H_1 \eta_1 + \gamma_2 H_2 \eta_2) \right]\right)_y = -\frac{\hat{w}_e}{\left(\frac{\partial Q_2}{\partial x}\right)_y}$$

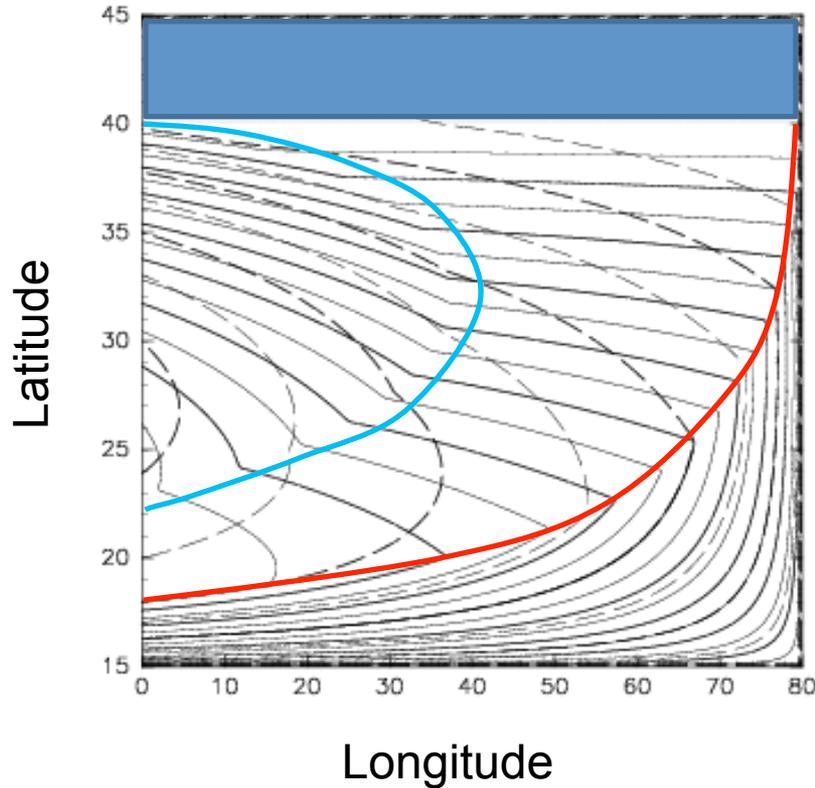
$$\frac{i\sigma(\eta_2 - \eta_1)}{\gamma_2 \left(\frac{\partial Q_2}{\partial x}\right)_y} + \left[ \left(\frac{\partial H_2}{\partial Q_2}\right)_y \left(\frac{\partial \eta_2 - \eta_1}{\partial y} \frac{1}{f}\right)_{Q_2} - \left(\frac{\partial}{\partial y} \eta_2\right)_{Q_2} \right] = 0$$

第1式はyを固定して、東岸から西に積分。第2式は Q2を固定して、東岸、もしくは、Outcropから南に積分。接続条件は

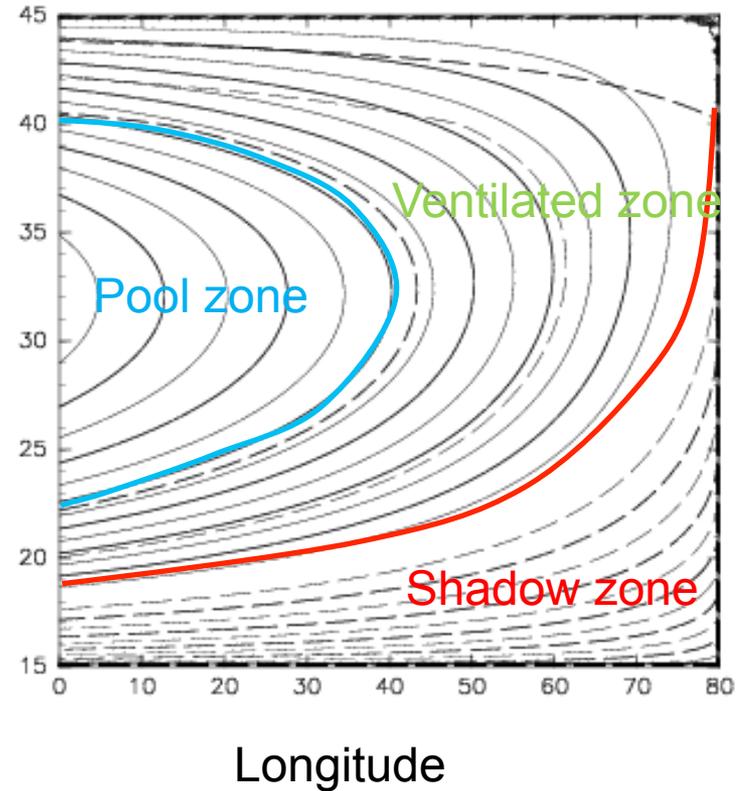
$$\xi = -\frac{q_2^{(s)}}{|\nabla Q_2^{(s)}|}, \quad \left(\frac{\partial \eta_2^{(v)}}{\partial y}\right)_{Q_2} = |\nabla Q_2^{(v)}| \left( \frac{if\sigma\xi}{\gamma_2 \left(\frac{\partial Q_2^{(v)}}{\partial x}\right)} + f_1 \left(\frac{\partial \xi}{\partial y}\right)_{Q_2} \right)$$

## 2.5層通気水温躍層

Outcropは40度、H2は東岸で750m、深さのコンタ間隔は25m



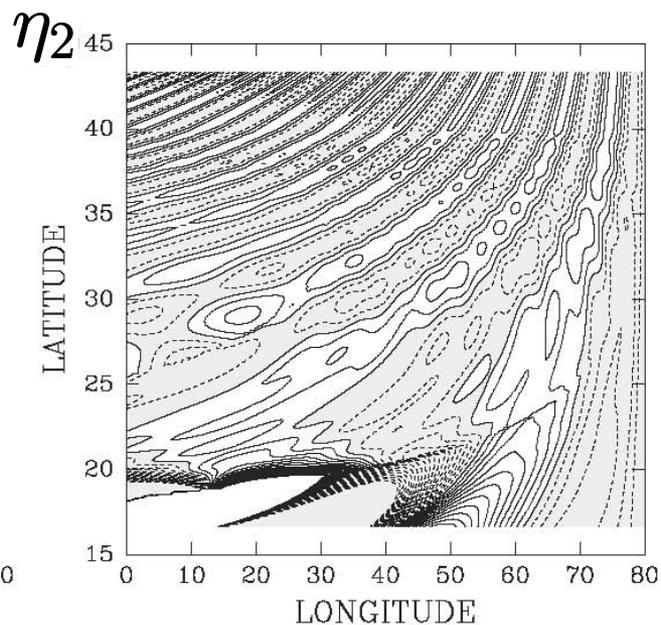
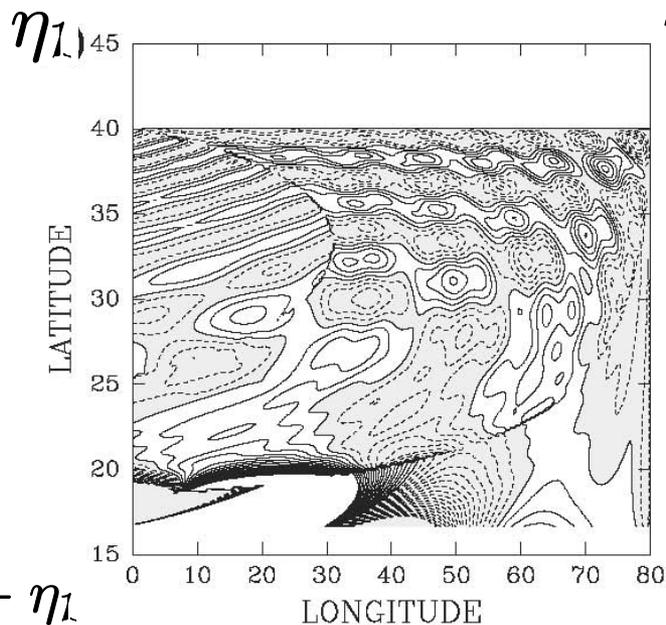
H1 : 上層の厚さ (実線)  
P1 : 上層の等圧線 (破線)



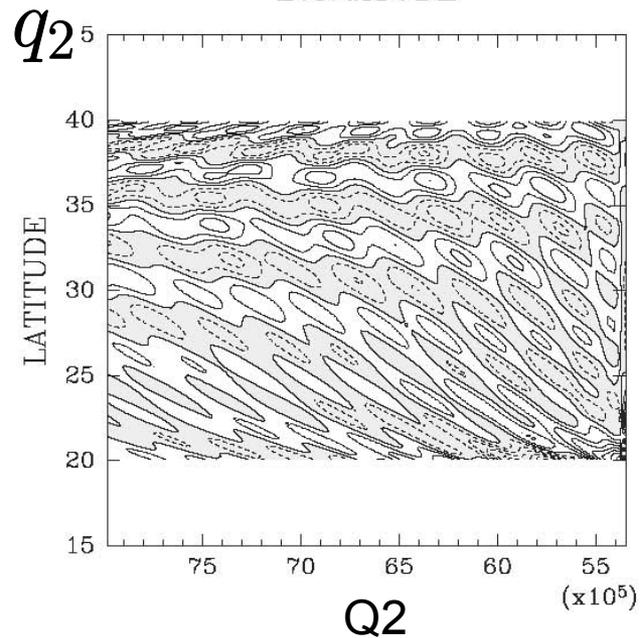
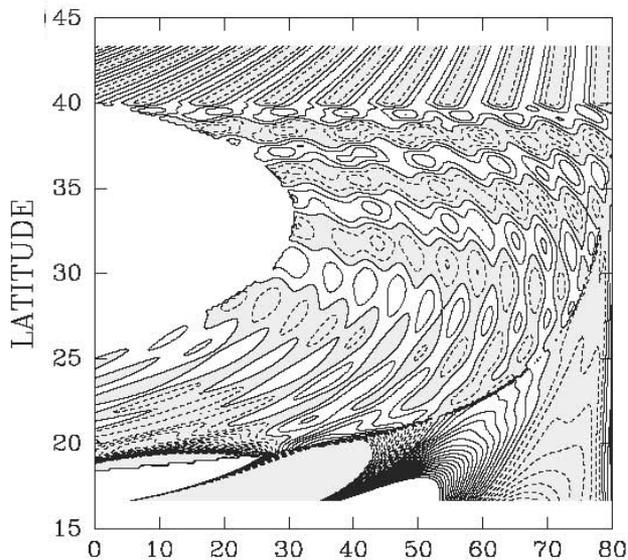
H2 : 2層目下面の深さ (実線)  
Q2 : 2層目のポテンシャル厚さ、  
(H2-H1) / f (破線)

解

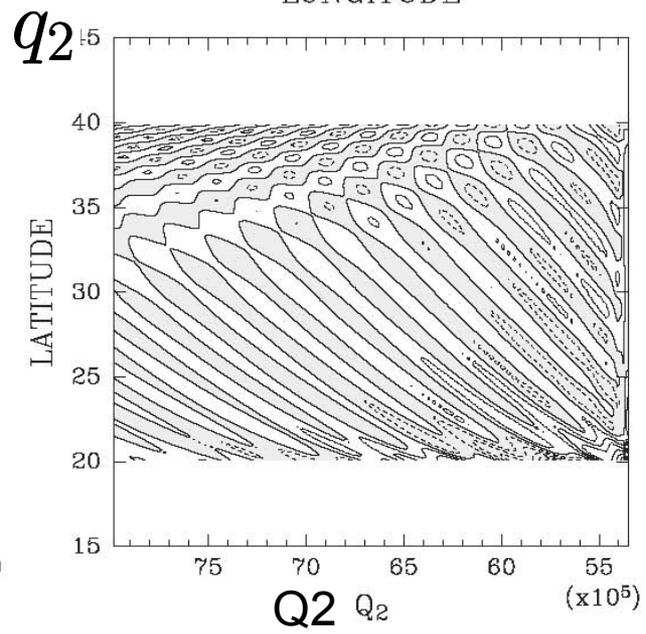
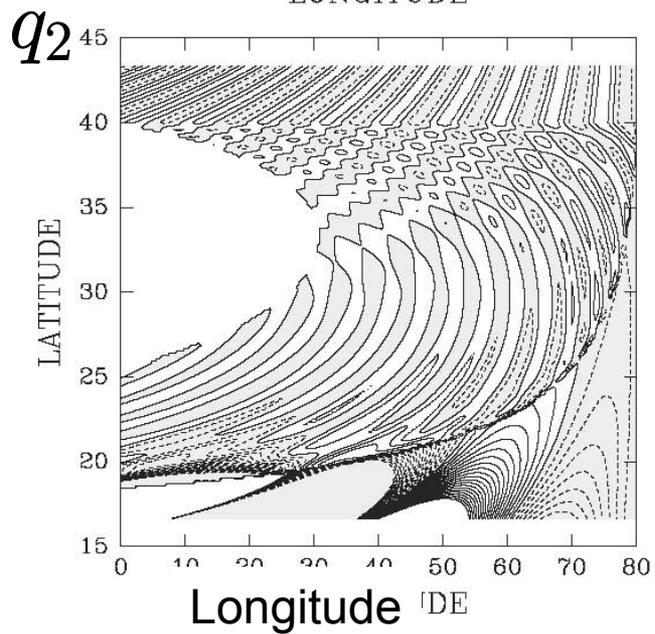
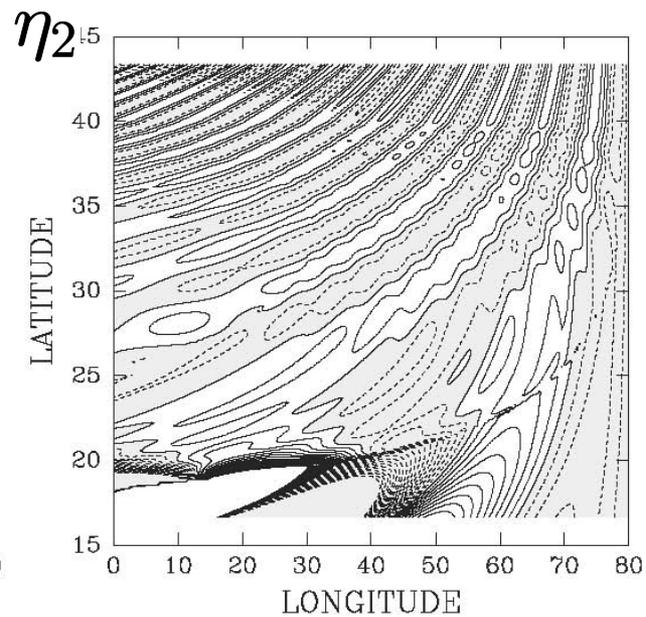
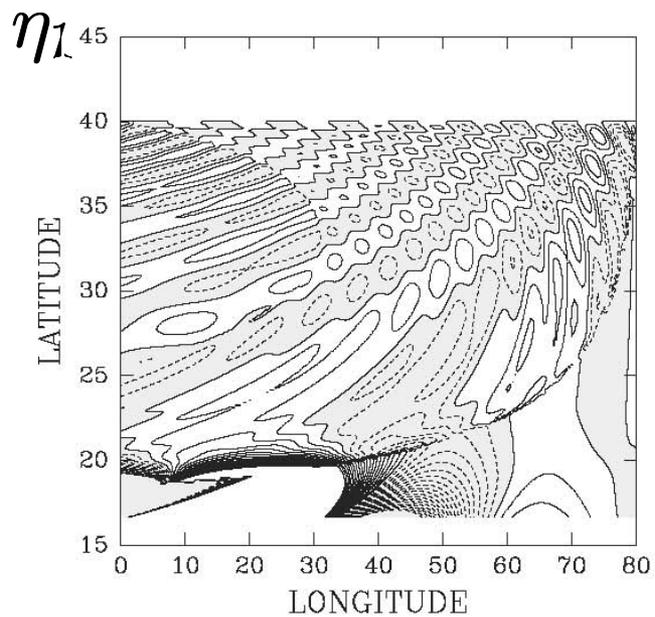
周期は2年、空間的には一様 $\hat{u}_e$



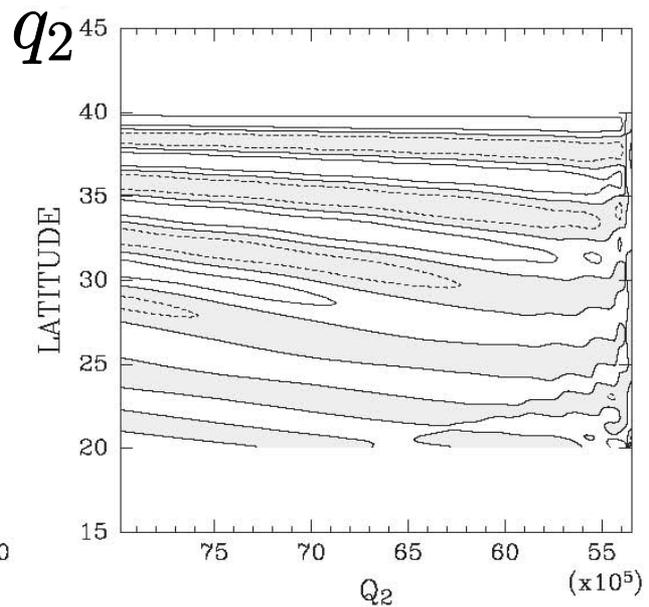
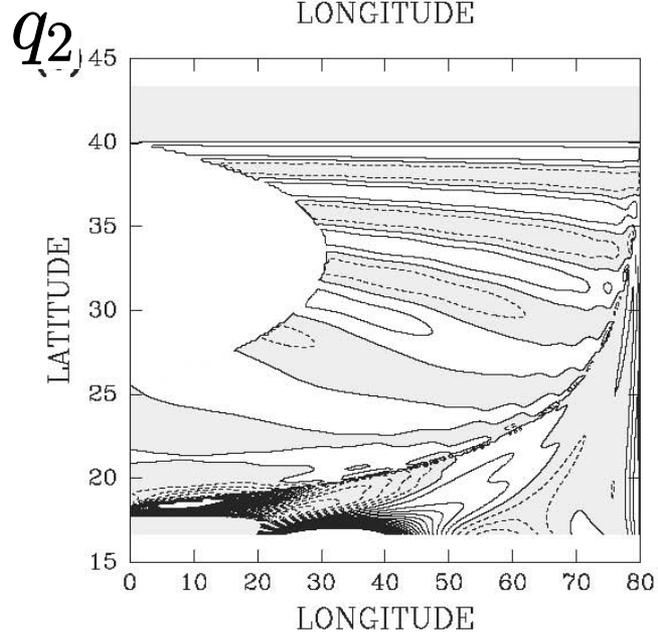
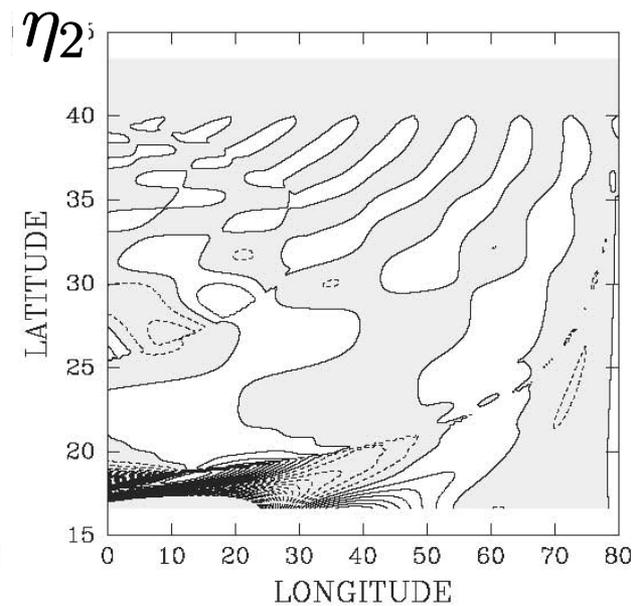
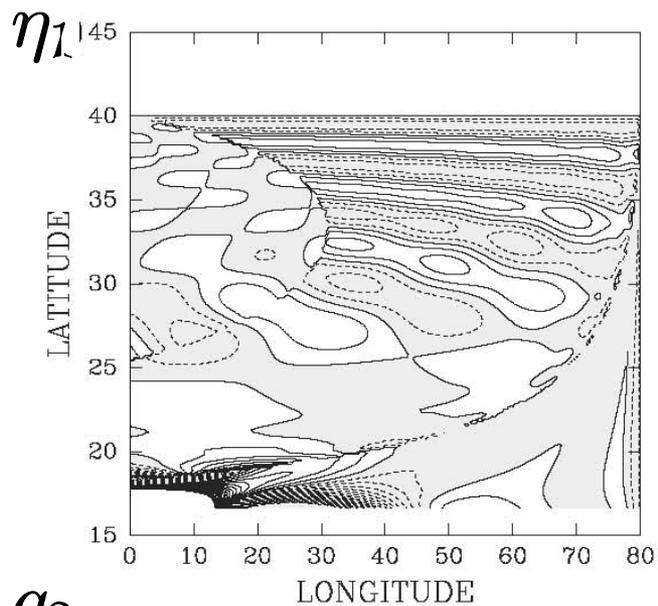
$$q_2 = \frac{\eta_2 - \eta_1}{f}$$



# 解: 東岸境界励起の波



解：東岸境界励起以外の風による波



# 波の性質(分散関係、群速度、鉛直構造)

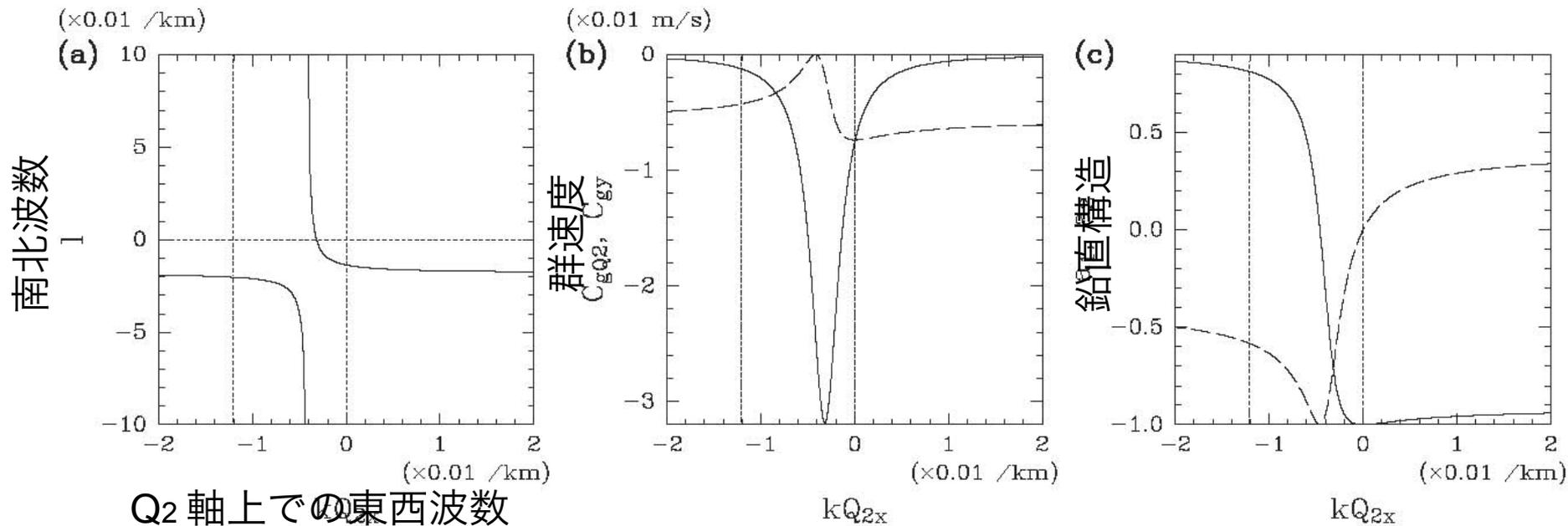
係数の空間依存性を無視した局所的な方程式。かつ、強制なし

$$i\sigma\eta_2 - \frac{\beta}{f^2} \left( \frac{\partial Q_2}{\partial x} \right) \left[ \gamma_1 H_1 \frac{\partial \eta_1}{\partial Q_2} + \gamma_2 H_2 \frac{\partial \eta_2}{\partial Q_2} \right] = 0$$

$$i\sigma(\eta_2 - \eta_1) + \gamma_2 \left( \frac{\partial Q_2}{\partial x} \right) \left[ \frac{f_1}{f} \frac{\partial}{\partial y} (\eta_2 - \eta_1) - \frac{\partial \eta_2}{\partial y} \right] = 0$$

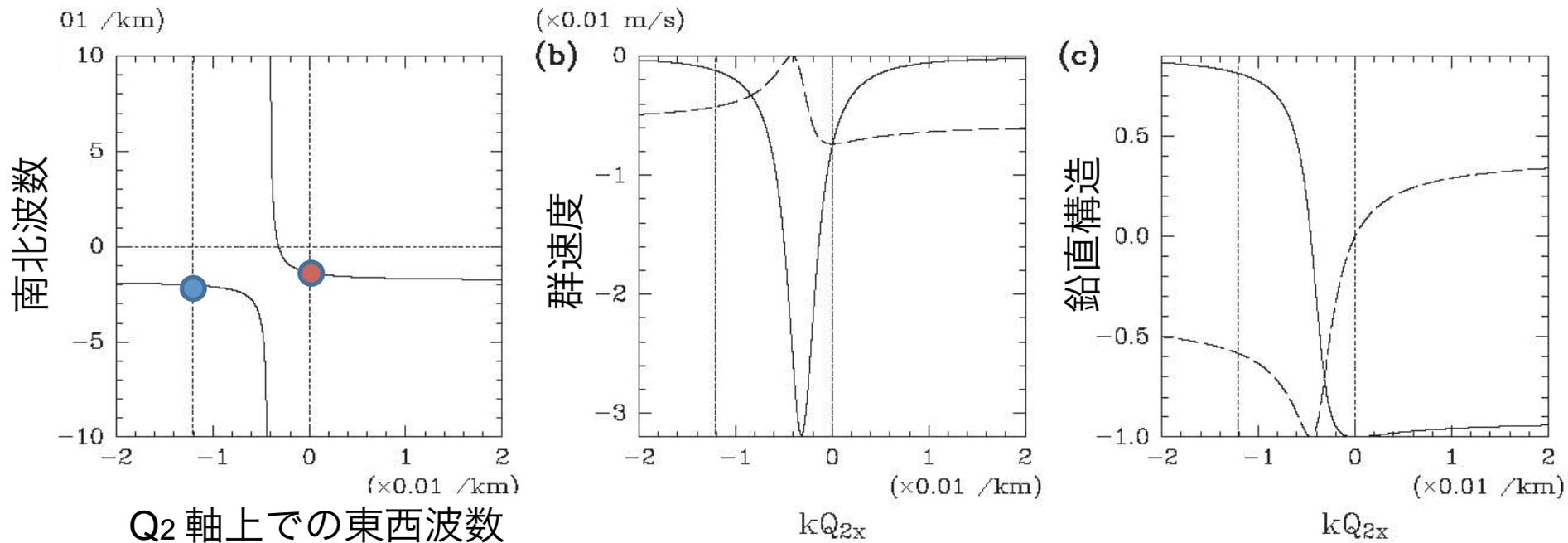
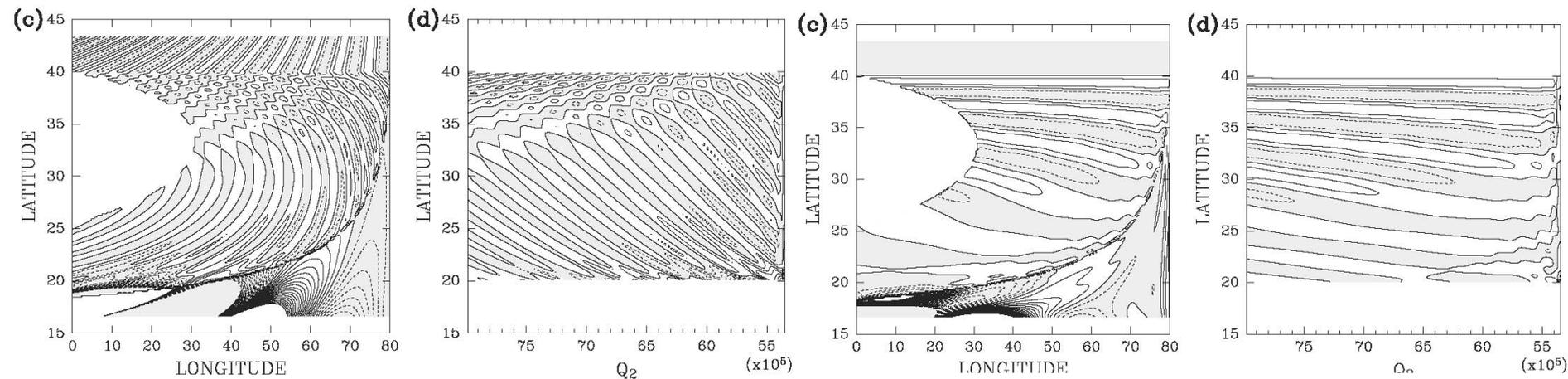
に  $(\eta_1, \eta_2) = (a_1, a_2)e^{-i(kx+ky)}$

を代入



経度60度、緯度30度での図

# 波の性質(分散関係、群速度、鉛直構造)



経度60度、緯度30度での図

# 波の発生機構

A-mode はoutcropで発生。Outcrop ( $y = y_1$ ) 近傍で方程式をたてる

$$i\sigma\eta_2 - \frac{\beta}{f^2} \left( \frac{\partial Q_2}{\partial x} \right) \left[ \gamma_1 H_1 \frac{\partial \eta_1}{\partial Q_2} + \gamma_2 H_2 \frac{\partial \eta_2}{\partial Q_2} \right] = -\hat{w}_e$$

$$i\sigma(\eta_2 - \eta_1) + \gamma_2 \left( \frac{\partial Q_2}{\partial x} \right) \left[ \frac{f_1}{f} \frac{\partial}{\partial y} (\eta_2 - \eta_1) - \frac{\partial \eta_2}{\partial y} \right] = 0$$

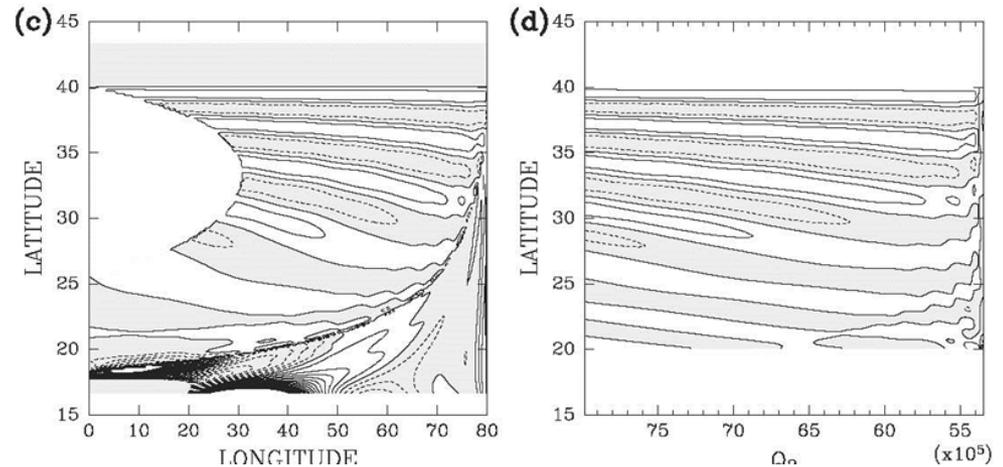
\* 外洋での風によるA-mode  
N-mode は存在しない

$$\eta_2 = i\hat{w}_e/\sigma$$

$$\frac{\partial \eta_1}{\partial y} + il_{A1} \left( \eta_1 + \frac{\hat{w}_e}{\sigma} \right) = 0$$

ここで、  $l_{A1} = \frac{f\sigma}{f_1\gamma_2 Q_{2,x}}$

境界条件  $\eta_1(x, y_1) = 0$   
は



より、 $\eta_1 = \frac{l_{A1}\hat{w}_e}{\sigma} \left[ 1 - e^{i f y_1 l_{A1} dy} \right]$

\* N-modeによるA-modeの励起

Outcrop近傍では  $H_1 \rightarrow 0$ ,  $f_1/f \rightarrow 1$   
となるので

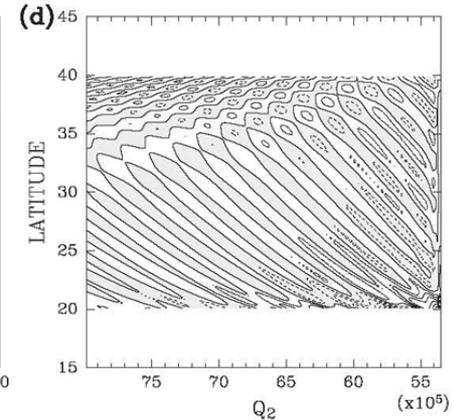
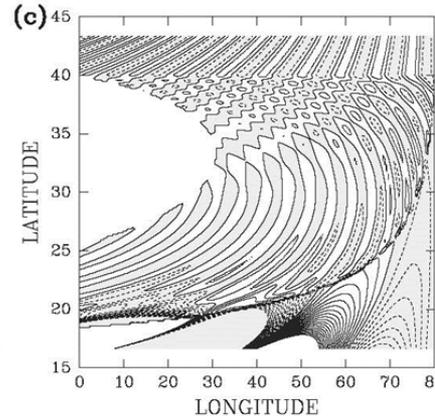
$$ik_N \eta_2 - \frac{\partial \eta_2}{\partial Q_2} = 0, \quad il_{A2}(\eta_2 - \eta_1) - \frac{\partial \eta_1}{\partial y} = 0$$

ここで  $k_N = \frac{\sigma f^2}{\gamma_2 \beta H_2 Q_{2,x}}$ ,  $l_{A2} = \frac{\sigma}{\gamma_2 Q_{2,x}}$

解は

$$\eta_2 = -\frac{i\hat{w}_e}{\sigma} e^{-i \int_{Q_{2E}}^{Q_2} k_N dQ_2} \quad \text{and}$$

$$\eta_1 = \frac{i\hat{w}_e}{\sigma} e^{-i \int_{Q_{2E}}^{Q_2} k_N dQ_2} [e^{-il_{A2}(y-y_1)} - 1] \quad \text{for } Q_2 > Q_{2E}.$$



境界条件

$$\eta_1 = \eta_2 = -i\hat{w}_w/\sigma \quad \text{at } Q_2 = Q_{2S}$$

$$\eta_1 = 0 \quad \text{at } y = y_1 \quad \text{for } Q_2 > Q_{2S}$$

## まとめ

- 周期的なエクマンポンピングに対する2.5層通気水温躍層の応答を調べた。
- 東岸起源の波 (N-mode) はshadow zoneとventilated zoneの境界を大きく変質することなく通過する
- Ventilated zone の変動は、東岸起源のN-mode、outcrop上の風によるA-mode、東岸起源のN-modeがoutcrop上を伝播することによるA-mode の三者からなる。
- outcrop上の風によるA-modeは東西波数がほぼゼロ、群速度は南西向きで、その東端は西に移動し、循環中央で、消える。
- N-mode に起源を持つA-modeは 波数は(Q<sub>2</sub>-y 面上で)南西向きで、outcropとshadow zone-ventilated zone境界をつなぐ形で伝播する。